University of Washington

Section 10.5 (Part 1) Method of separation of variables

Settings :
Partial differential equation: Given a rod of length L, the variation of temperature u at time t (in s) and at the position x (in cm) in the rod is governed by the partial differential equation

$$\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

where α is called the thermal diffusivity.
Initial conditions: Usually given

- the distribution of temperature at $t = 0$: $u(0, x) = f(x)$.
- Conditions on the boundaries of the rod:
  - either a constant temperature on the boundaries:

    $$u(t, 0) = u(t, L) = 0$$

  - or insulated boundaries

    $$\frac{\partial u}{\partial x}(t, 0) = \frac{\partial u}{\partial x}(t, L) = 0$$

Problem: Can we find the evolution of the temperature?

Exercise 1. Use the method of separation of variable to solve the heat flow problem

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < L, \quad 0 < t,$$

$$u(0, x) = 3\sin\left(\frac{2\pi x}{L}\right) - 6\sin\left(\frac{2\pi x}{L}\right), \qquad 0 < x < L$$

$$u(t, 0) = u(t, L) = 0$$

Exercise 2. Determine whether the method of separation of variables can be used to replace the given partial differential equation into a pair of ordinary differential equations.

1. $x\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = 0$
2. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial t} = 0$