
Section 10.4

Definition:

- A function f is odd if
 - for all x in the domain $(-x)$ is in the domain.
 - for any x in the domain of f , $f(-x) = -f(x)$.
- A function f is even if
 - for all x in the domain $(-x)$ is in the domain.
 - for any x in the domain of f , $f(-x) = f(x)$.

Theorem: Given a $2L$ -periodic function f ,

- If f is an even function, all the coefficients b_n are zero and the Fourier series is a cosine series

$$\hat{f}(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

with

$$a_n = \frac{2}{L} \int_0^L f(t) \cos\left(\frac{n\pi t}{L}\right) dt$$

- If f is an odd function, all the coefficients a_n are zero. The Fourier series of f is a sine series

$$\hat{f}(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

with

$$b_n = \frac{2}{L} \int_0^L f(t) \sin\left(\frac{n\pi t}{L}\right) dt$$

Exercise 1. Given the function $f(x) = x$ for $0 \leq x \leq 1$.

1. Find a sine Fourier series that is convergent to f on the interval $(0, 1)$.

Sketch the graph of the function to which the Fourier series converges over 3 periods.

2. Find a cosine Fourier series that is convergent to f on the interval $(0, 1)$.

Sketch the graph of the function to which the Fourier series converges over 3 periods.