Section 7.1 System of linear differential equations

Exercise 1. Two large tanks, each containing 100 gal of brine solution, are interconnected by pipes. Fresh water flows into tank A at a rate of 6 gal/min, and fluid is drained out of tank B at the same rate. 8 gal/min of fluid are pumped from tank A to tank B, and 2 gal/min are pumped from tank B to tank A. The initial amount of salt in tank A is A_0 and the initial amount of salt in tank B is B_0. The liquid inside each tank is well stirred so that each mixture is homogeneous.

Find a system of 1st order differential equations satisfied by the amount of salt in each tank. Express the system in matrix notation.

Exercise 2. On a smooth surface, a mass $m_1 = 2kg$ is attached to a fixed wall by a spring with spring constant $k_1 = 4$ N/m. Another mass $m_2 = 1$ kg is attached to the first object by a spring with spring constant $k_2 = 2$ N/m. The objects are aligned horizontally so that the springs are in their natural lengths. If both objects are displaced 3 m to the right of their equilibrium positions and released, what are the equations of motion for the two objects?

Express the problem in terms of a system of 1st order differential equations.

Exercise 3. Where a higher order differential equation leads to a system of first order differential equations Transform the differential equation into a system of first order differential equations.

$$y^{(4)} - 2y'' - 5y = \cos t$$

Exercise 4. Where a system of first order differential equations leads to a higher order differential equations Consider the system of differential equations

$$x_1'(t) = -2x_1(t) + x_2(t), \qquad x_2'(t) = x_1(t) - 2x_2(t)$$

1. Solve the first equation for x_2 and substitute into the second equation, thereby obtaining a second order equation for x_1, Solve this equation for x_1 and then determine x_2.

2. Find the solution of the given system that also satisfies the initial conditions $x_1(0) = 2, x_2(0) = 3$.