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Section 7.3

1 Determinant and systems of equations

Exercise 1. Find the determinant of

$$A = \begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 4 & 1 \\ 2 & 4 & 0 \\ -4 & -8 & 1 \end{pmatrix}$$

Definition: A square matrix is invertible if there exists a matrix B such that $AB = BA = I$.
The matrix B is called the inverse of A and is written A^{-1}.

Theorem: A matrix A is invertible if and only if $\det(A) \neq 0$.

Exercise 2. Solve the system of equations

$$\begin{cases} x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + 2x_3 = -1 \end{cases}$$

2 Linear independence

Definition: The vectors X_1, $X_2, \cdots, X_n$ are linearly independent if the only linear combination of $c_1X_1 + c_2X_2 + \cdots + c_nX_n = 0$ is for $c_1 = c_2 = \cdots = c_n = 0$.

Remark:

Exercise 3. Are the vectors $X_1 = (1, 1, 0)$, $X_2 = (0, 1, 1)$ and $X_3 = (1, 0, -1)$ linearly independent.

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Theorem: n vectors of $\mathbb{R}^n$, X_1, $X_2, \cdots, X_n$ are linearly independent if $\det(X_1, X_2, ..., X_n) \neq 0$.

Remark:

Exercise 4. Are the vector functions

$$X_1(t) = \begin{pmatrix} e^{-t} \\ 2e^{-t} \end{pmatrix}, \qquad X_2(t) = \begin{pmatrix} e^{-t} \\ e^{-t} \end{pmatrix}, \qquad X_3(t) = \begin{pmatrix} 3e^{t} \\ 0 \end{pmatrix}$$

linearly independent?

3 Eigenvalues, Eigenvectors

Definition: Let A be a square matrix and I be the identity matrix. r is an eigenvalue if $\det(A - rI) = 0$.
A non zero vector X such that $AX = rX$ is called an eigenvector.

Exercise 5. Find the eigenvalues and eigenvectors of

$$A = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix}$$