# Section 7.6

**Exercise 1.** Given the systems

$$X' = \begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix} X$$

$$X' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} X.$$

1. Find the general solution of the given system of equation in terms of real valued functions.
2. Describe the behavior of the solution as $t \to \infty$.

**Theorem:** Given a system of differential equations

$$X'(t) = AX(t).$$

If $r_1 = \lambda + i\mu$, and $r_2 = \lambda - i\mu$ is a pair of complex conjugate eigenvalues and $\xi = \mathbf{a} \pm i\mathbf{b}$ a corresponding pair of eigenvectors. Then the vectors

$$\mathbf{u} = e^{\lambda t}(\mathbf{a}\cos\mu t - \mathbf{b}\sin\mu t), \qquad \mathbf{v} = e^{\lambda t}(\mathbf{a}\sin\mu t + \mathbf{b}\cos\mu t)$$

are real values solutions of the system.

**Exercise 2.** Find the general solution of the given system of equations in term of real valued functions

$$X' = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} X$$

**Exercise 3.** For the system

$$X' = \begin{pmatrix} \alpha & 1 \\ -1 & \alpha \end{pmatrix} X$$

1. Determine the eigenvalues in terms of $\alpha$.
2. Find the critical value or values of $\alpha$ where the qualitative nature of the phase portrait for the system changes.
3. Draw a phase portrait for a value slightly below and for another value slightly above each critical value.