University of Washington

Section 7.8

Exercise 1. Consider the system

$$X'(t) = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} X(t)$$

Find the general solution of the system.

Describe how the trajectories behave as $t \to \infty$.

Exercise 2. Solve the initial value problem

$$X'(t) = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} X(t), \qquad X(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

Exercise 3. Consider the system

$$X'(t) = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 2 & 2 \\ 4 & 2 & 3 \end{pmatrix} X(t).$$

Find the general solution of the system.

Method for solving system with 1 eigenvalue with multiplicity 2:
Find the space of eigenvectors for the eigenvalue λ.
2 cases:

- **First case:** There exists 2 linearly independent eigenvectors V_1 and V_2. Then

  $$X_1(t) = e^{\lambda t} V_1 \quad \text{and} \quad X_2(t) = e^{\lambda t} V_2$$

  are two linearly independent solutions.

- **Second case:** There is no pair of linearly independent eigenvectors.

  - Find an eigenvector V_1.
  - Find V_2 a generalized eigenvector

  $$(A - \lambda I)V_2 = V_1$$

  Then

  $$X_1 = e^{\lambda t} V_1 \quad \text{and} \quad X_2 = e^{\lambda t}(tV_1 + V_2)$$

  are two linearly independent solutions

Exercise 4. For the system

$$X' = \begin{pmatrix} 2 & -5 \\ \alpha & -2 \end{pmatrix} X$$

1. Determine the eigenvalues in terms of α.
2. Find the critical value or values of α where the qualitative nature of the phase portrait for the system changes.
3. Draw a phase portrait for a value slightly below and for another value slightly above each critical value.