Texas A&M University

Recitation 2
Sections 6.5, 7.1 - solutions

1. Evaluate $\int (2+3\,\mathrm{e}^x)^7\,\mathrm{e}^x\,\mathrm{d}x$.

   Answer. `Substitution` $u = 2 + 3\,\mathrm{e}^x$.

$$\boxed{\int (2+3\,\mathrm{e}^x)^7\,\mathrm{e}^x\,\mathrm{d}x = \frac{1}{24}\,(2+3\,\mathrm{e}^x)^8 + C}$$

2. Find $\int x^2\sqrt{x^3+5}\mathrm{d}x$.

   Answer. `Substitution` $u = x^3 + 5$.

$$\boxed{\int x^2\sqrt{x^3+5}\mathrm{d}x = \frac{2}{9}(x^3+5)^{3/2}}$$

3. Find $\int_0^3 \left(\mathrm{e}^{x^2+1}\right) x\mathrm{d}x$.

   Answer. `Substitution` $u = x^2 + 1$, $\mathrm{d}u = 2x\mathrm{d}x$.

   `If` $x = 0$, $u = 1$. `If` $x = 3$, $u = 10$.

   $\int_0^3 \left(\mathrm{e}^{x^2+1}\right) x\mathrm{d}x = \frac{1}{2}\int_1^{10} \mathrm{e}^u\,\mathrm{d}u$.

$$\boxed{\int_0^3 \left(\mathrm{e}^{x^2+1}\right) x\mathrm{d}x = \frac{1}{2}\left(\mathrm{e}^{10} - \mathrm{e}\right)}$$

4. Find $\int 4r\sqrt{3+4r}\mathrm{d}r$.

   Answer. `Substitution` $u = 3 + 4r$. $\mathrm{d}u = 4\mathrm{d}r$. $4r = u - 3$.

   $\int 4r\sqrt{3+4r}\mathrm{d}r = \frac{1}{4}\int (u-3)\sqrt{u}\mathrm{d}u = \frac{1}{4}\left(\frac{2u^{(5/2)}}{5} - 2u^{(3/2)}\right) + C$.

$$\boxed{\int 4r\sqrt{3+4r}\mathrm{d}r = \frac{(3+4r)^{(5/2)}}{10} - \frac{(3+4r)^{(3/2}}{2} + C}$$

5. Find $\int_0^{\pi/3} (\cos 2t + 1)^4 \sin 2t\mathrm{d}t$.

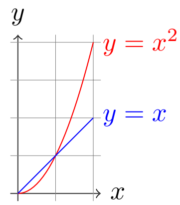
   Answer. `Substitution` $u = 1 + \cos 2t$. $\mathrm{d}u = -2\sin(2t)\mathrm{d}t$.

   `If` $t = 0$, $u = 2$. `If` $t = \frac{\pi}{3}$, $u = 1 - \frac{1}{2} = \frac{1}{2}$.

$$\int_0^{\pi/3} (\cos 2t + 1)^4 \sin 2t\mathrm{d}t = -\frac{1}{2}\int_2^{1/2} u^4 \mathrm{d}u = \frac{1}{10}(2^5 - \frac{1}{2^5})$$

$$\boxed{\int_0^{\pi/3} (\cos 2t + 1)^4 \sin 2t\mathrm{d}t = \frac{1023}{320}}$$

6. Find the area of the region between the curves $y = x^2$ and $y = x$ over the interval $[0, 2]$.

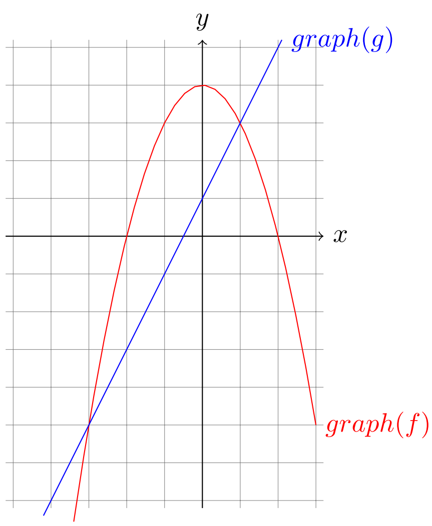


Answer. The curves crosses at $x = 1$.

$$\mathcal{A} = \int_0^1 x - x^2 \mathrm{d}x + \int_1^2 x^2 - x\mathrm{d}x = \frac{1}{2} - \frac{1}{3} + \frac{7}{3} - \frac{3}{2}$$

$$\boxed{\mathcal{A} = 1}$$

7. Find the area of the region bounded by the graphs of the function $f(x) = 4 - x^2$ and $g(x) = 2x + 1$



Answer. The graphs intersect at x solution of $f(x) = g(x)$.

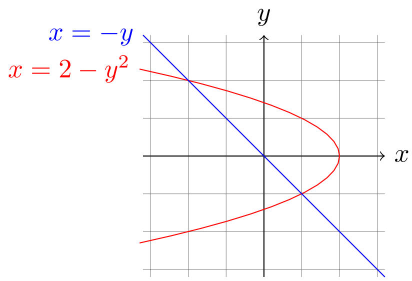
$4 - x^2 = 2x + 1$ iff $x^2 + 2x - 3 = 0$, iff $x = 1$ or $x = -3$.

2 intersection points $x = 1$ and $x = -3$.

$$\mathcal{A} = \int_{-3}^1 f(x) - g(x)\mathrm{d}x = \int_{-3}^1 3 - 2x - x^2 \mathrm{d}x$$

$$\boxed{\mathcal{A} = \frac{32}{3}}$$

8. Find the area of the region bounded by the curves $x + y^2 = 2$ and $x + y = 0$.



Answer. x `can be easily express as a function of` y. `Therefore we will consider the variable` y.

`The intersection points are solutions of` $-y = 2 - y^2 = x$.

`2 solutions :` $y = -1$ `and` $y = 2$.

$\mathcal{A} = \int_{-1}^{2} (2 - y^2) - (-y) \mathrm{d}y$

$$\boxed{\mathcal{A} = \frac{9}{2}}$$