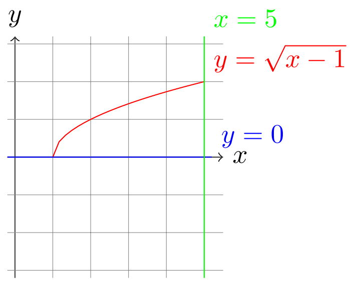
Recitation 3
Sections 7.2,7.3

1. Set up **and** evaluate an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = \sqrt{x-1}$, $y = 0$, $x = 5$ about the x-axis.

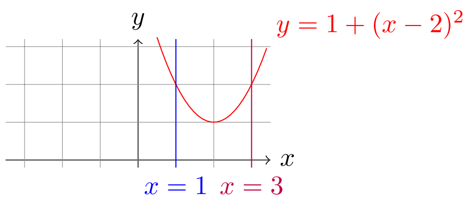


Answer. `Variable of integration:` x. `Washer method:`

$$V = \int_1^5 \pi(\sqrt{x-1})^2 \mathrm{d}x = \pi \int_1^5 (x-1)\mathrm{d}x$$

$$\boxed{V = \int_1^5 \pi(\sqrt{x-1})^2 \mathrm{d}x = 8\pi}$$

2. Set up **and** evaluate an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = 1 + (x-2)^2$, $y = 0$, $x = 1$, and $x = 3$ about the y-axis.

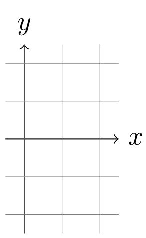


Answer. `Variable of integration` x. `Cylindric shells method:`

$$V = \int_1^3 2\pi x(1 + (x-2)^2)\mathrm{d}x$$

$$\boxed{V = \int_1^3 2\pi x(1 + (x-2)^2)\mathrm{d}x = \frac{32\pi}{3}}$$

3. Set up **and** evaluate an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = \dfrac{1}{x}$, $y = x$, and $x = 2$ about the x-axis.

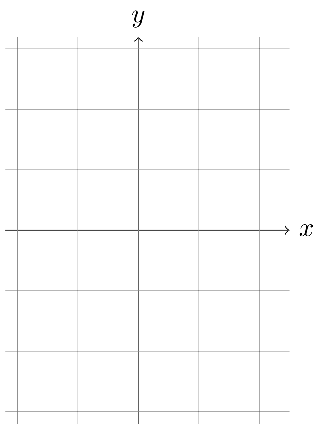


Answer.

See Exercise 10, review 1

4. Set up **and** evaluate an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = x^3$ and $y = x^2$

   (a) about the line $y = 1$.



   (b) about the line $x = -1$.
   Answer.

   See Exercise 11, review 1

5. Set up **and** evaluate an integral for the volume of the solid obtained by rotating the area bounded by the curves $y = \cos x^2$, $y = 0$, $x = \sqrt{\frac{\pi}{3}}$, and $x = \sqrt{\frac{\pi}{2}}$ about the y-axis.

   Answer. `Variable` x. `Washer method.`

   $$V = \int_{\sqrt{\frac{\pi}{3}}}^{\sqrt{\frac{\pi}{2}}} 2\pi x \cos(x^2)\mathrm{d}x$$

   `Substitution` $u = x^2$.

   $$\boxed{V = \int_{\sqrt{\frac{\pi}{3}}}^{\sqrt{\frac{\pi}{2}}} 2\pi x \cos(x^2)\mathrm{d}x = \pi(1 - \frac{\sqrt{3}}{2})}$$

6. Set up **and** evaluate an integral for the volume of the solid obtained by rotating the area bounded by the curves $x = 4 - y^2$, $x = 0$ about the y-axis.

   Answer. `Variable` y. `Washer method.`

   `2 intersection points` $y = -2$ `and` $y = 2$.

   $$V = \int_{-2}^{2} \pi(4 - y^2)^2 \mathrm{d}y$$

   $$\boxed{V = \int_{-2}^{2} \pi(4 - y^2)^2 \mathrm{d}y = \frac{512\pi}{15}}$$