Review for Exam 1
Sections 6.4-7.5

1 Section 6.4

Exercise 1. Find the derivative of $\int_0^{\cos x} t^2 - 5 \mathrm{d}t$

Exercise 2. Find $\int_{-1}^{2} 1 - |x| \, \mathrm{d}x$.

Exercise 3. Find the derivative of $f(x) = \int_{-x}^{x} t^2 + 1 \; dt$ at $x = 1$, at $x = 0$.

2 Section 6.5

Exercise 4. Find the following integrals

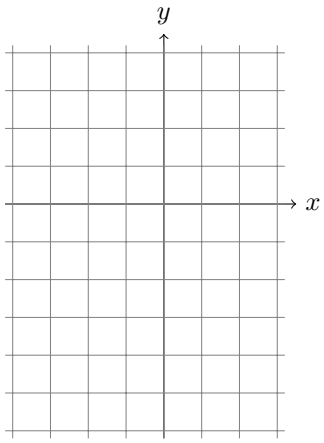
1. Find $\int 4x\sqrt{2x-3}\mathrm{d}x$.
2. $\int \cos x(\sin x + 1)^2 \mathrm{d}x$.
3. $\int 6x \, \mathrm{e}^{x^2} \, \mathrm{d}x$.
4. $\int_{\sqrt{2}/2}^{1/2} \frac{1}{\sqrt{1-x^2}} \mathrm{d}x$.
5. $\int_0^{\pi} \sin^{10} x \cos^3 x \mathrm{d}x$.
6. $\int_0^{2} 6x^2\sqrt{x^3+1}\mathrm{d}x$.
7. $\int_0^{\pi/12} \tan 3x \mathrm{d}x$.
8. $\int (2 + 3\,\mathrm{e}^x)^7 \, \mathrm{e}^x \, \mathrm{d}x$.
9. $\int \cos(5x-6)\mathrm{d}x$.
10. $\int_0^{3} \mathrm{e}^{x^3+1} \, x^2 \mathrm{d}x$.

11. $\displaystyle\int_0^{\pi/2} \frac{\sin x}{2+\cos x}\mathrm{d}x.$
12. $\displaystyle\int_0^{1} \frac{x}{\sqrt{1-x^2}}\mathrm{d}x$
13. $\displaystyle\int t^2\sqrt{t^3+5}\mathrm{d}t.$
14. $\displaystyle\int_2^3 \frac{x^2}{x^3-1}\mathrm{d}x.$
15. $\displaystyle\int 18x^2(6x^3+7)^{(1/2)}\mathrm{d}x.$
16. $\displaystyle\int \frac{16u+7}{8u^2+7u}\mathrm{d}u.$
17. $\displaystyle\int_{1/\pi}^{(\mathrm{e}^4)/\pi} \frac{\ln(\pi t)}{t}\mathrm{d}t.$
18. $\displaystyle\int_1^3 x\sqrt{x-1}\mathrm{d}x.$
19. $\displaystyle\int_0^5 x(x-3)^6\mathrm{d}x.$

3 Section 7.1

Exercise 5. Find the area between the curves

1. $y=x^3$ and $y=x$ over $[0,2]$.
2. $f(x)=4-x^2$ and $g(x)=2x+1$



3. $f(x)=x^3$ and $g(x)=x^2$ between $x=0$ and $x=4$.

4 Sections 7.2, 7.3

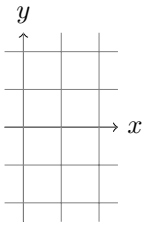
Exercise 6. (Fall 2011) Find the volume of the solid formed by rotating the region bounded by $x = 0$, $y = \ln x$, $y = 0$, $y = 2$ about the y-axis.

Exercise 7. The region bounded by $y = 2x - x^2$ and $y = 0$ is revolved around the y-axis. Find the volume.

Exercise 8. Using cylindric shells, set up an integral for the volume of the solid formed by rotating the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the line $y = -1$.

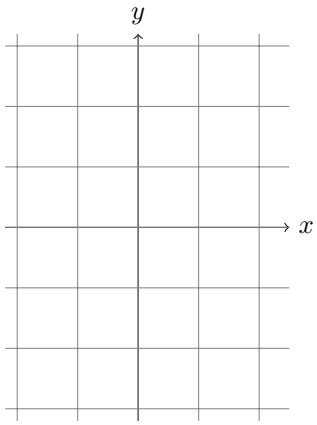
Exercise 9. (Fall 2011) Find the volume of the solid whose base is the area enclosed by $y = \sin x$ and $y = \cos x$ from $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ with cross sections perpendicular to the x-axis that are squares.

Exercise 10. Find the volume of the solid obtained by rotating the region bounded by the curves $y = \dfrac{1}{x}$, $y = x$, and $x = 2$ about the x-axis.



Exercise 11. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^3$ and $y = x^2$

1. about the line $y = 1$.



2. about the line $x = -1$.

Exercise 12. Let $\mathcal{S}$ be a solid whose base is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(0, 2)$, and whose cross sections perpendicular to the y-axis are semicircles. Compute the volume of $\mathcal{S}$.

5 Section 7.4

Exercise 13. (4p448) When a particle is a a distance x meters from the origin, a force of $\cos(\pi x/3)$ Newtons acts on it. How much work is done in moving the particle from $x = 1$ to $x = 2$?

Exercise 14. A 15N weight is suspended vertically at the end of a 30m long rope. The rope weight 6N. How much work is required to pull the weight to the top?

Exercise 15. A rope 20 feet long weighing 2 pounds per foot supports a 160lb weight on the side of the building. How much work in (ft-lb) is required to pull the weight to the top of the building?

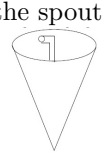
Exercise 16. (8p448) If the work to stretch a spring 1 ft beyond its natural length is 12 ft-lb, how much work is needed to stretch it 9 in. beyond its natural length?

Exercise 17. A force of 10lb is required to hold a spring stretched 1/3 ft beyond its natural length. How much work is done in stretching it from its natural length to 1/2 ft beyond its natural length?

Exercise 18.

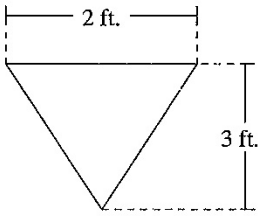
(10 points) A cylindrical stock tank has height $h = \frac{2}{3}$ m. The diameter is $d = 1$ m; illustration below. The tank is full of liquid (density $= \rho$ kg/m^3). What is the work required to pump all the liquid out of the top of the stock tank? (Leave your answer in terms of ρ and g the gravitational constant.)

Exercise 19. The conical tank shown below is 3 feet tall (not including the spout), has a 2 foot radius at the top, is full of water (density ρg), and has a 1 foot tall spout. Find the work required to pump all the water out of the spout. Leave your answer in terms of ρ and g.



Exercise 20.

Consider a trough full of liquid with weight density ρg, where the trough is 8 ft. long and its cross-section is given by the figure



Calculate the work needed to pump all of the fluid over the top. (10 points)

6 Section 7.5

Exercise 21. Find the average value of the following functions

1. $g(x) = \dfrac{3}{x}$ over $[1, 3]$.
2. $j(x) = x^2$ over $[1, 4]$.
3. (Fall 2011) $f(x) = \cos^2 x \sin(2x)$ over the interval $\left[0, \dfrac{\pi}{2}\right]$.
4. $f(x) = \dfrac{x}{\sqrt{x+1}}$ on the interval $[0, 3]$.