Review for Exam 2
Sections 8.1-9.4

1 Section 8.1

Exercise 1.

1. $\int x\cos(3x)\mathrm{d}x$
2. $\int (x^2+x)\sin x\mathrm{d}x.$
3. $\int x\cos^3 x\mathrm{d}x.$
4. $\int x^2\,\mathrm{e}^{-x}\,\mathrm{d}x.$
5. $\int x^2\ln(x)\mathrm{d}x.$
6. $\int_1^2 \ln(u)\mathrm{d}u.$
7. $\int_0^{\pi/2} (x+2)\cos x\mathrm{d}x.$
8. $\int_0^1 (x^2-x)\,\mathrm{e}^{2x}\,\mathrm{d}x.$
9. $\int_0^1 x\,\mathrm{e}^{-x^2}\,\mathrm{d}x.$
10. $\int_0^{\pi/4} \tan x\mathrm{d}x.$

2 Section 8.2

Exercise 2. Evaluate

1. $\int \sin^2 x\cos x - 2\cos^4 x\sin x\mathrm{d}x.$
2. $\int \sin^2 t\cos^2 t\mathrm{d}t$
3. $\int \sin t\cos t\mathrm{d}t.$

4. $\int \sin^4 t \mathrm{d}t$.
5. $\int \cos(5t)\sin(3t)\mathrm{d}t$.
6. $\int t\cos(5t)\sin(3t)\mathrm{d}t$.
7. $\int_0^{\pi/4} \frac{1+\sin x}{\cos^2 x}\mathrm{d}x$.

3 Section 8.3

1. $\int_0^1 \frac{u}{\sqrt{1-u^2}}\mathrm{d}u$.
2. $\int_0^1 \sqrt{1-x^2}\mathrm{d}x$.
3. $\int_0^4 \frac{1}{(16+t^2)^2}\mathrm{d}t$.
4. $\int \frac{x^4}{\sqrt{1+x^2}}\mathrm{d}x$. Change it to $\int \frac{x^3}{\sqrt{1+x^2}}\mathrm{d}x$, or $\int \frac{1}{\sqrt{1+x^2}}\mathrm{d}x$.

4 Sections 8.4

Exercise 3. Find the partial fraction decomposition and evaluate the integral of

1. $\frac{3x^2+12x+11}{(x+1)(x+2)(x+3)}$.
2. $\frac{x}{(x-1)(x^2+1)}$.
3. $\frac{x^4}{(x-1)(x+1)}$.
4. $\frac{x^2-3x-2}{(x^2-1)(x-1)}$.
5. $\frac{3-2x}{x^2-3x+2}$.
6. $\frac{x^4}{(x-1)^2(x^2+4)}$.

5 Section 8.9

Exercise 4. Evaluate the following integrals of show that they are divergent.

1. $\int_2^6 \frac{x}{\sqrt{x-2}}\mathrm{d}x$.

2. $\displaystyle\int_0^\infty \frac{1}{(x-3)^2}\mathrm{d}x$
3. $\displaystyle\int_1^{\mathrm{e}} \frac{1}{t\sqrt{\ln t}}\mathrm{d}t.$
4. $\displaystyle\int_1^2 \frac{\sqrt{x^2-1}}{x}\mathrm{d}x.$
5. $\displaystyle\int_0^\infty \frac{1}{(x+1)^2(x+2)}\mathrm{d}x.$
6. $\displaystyle\int_1^\infty \frac{\operatorname{Arctan} x}{x^2}\mathrm{d}x.$

Exercise 5. Use the comparison theorem to prove that

1. $\displaystyle\int_2^\infty \frac{x^2-2}{x^5+3}\mathrm{d}x$ is convergent.
2. $\displaystyle\int_1^\infty \frac{\sin^2 x}{x^3+5x}\mathrm{d}x$ is convergent.
3. $\displaystyle\int_0^1 x\cos^4\left(\frac{1}{x}\right)\mathrm{d}x$ is convergent.
4. $\displaystyle\int_1^\infty \frac{\ln(x+1)}{\sqrt{x}}$ is divergent.
5. $\displaystyle\int_0^1 \frac{x^2+3}{1-x}\mathrm{d}x$ is divergent.

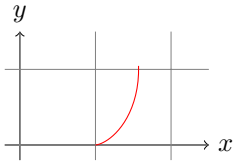
6 Section 9.3

Exercise 6. Find the length of the curve

$$x(t)=\cos^3 t,\quad y(t)=\sin^3 t,\quad t\in[0,\pi/2].$$

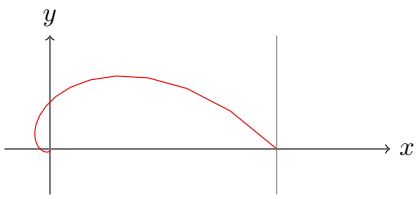
Exercise 7. (exam 2 2010) Find the length of the curves

$$x(\theta)=\cos\theta+\theta\sin\theta,\quad y(\theta)=\sin\theta-\theta\cos\theta\quad 0\leqslant\theta\leqslant\pi/2.$$



Exercise 8. Find the length of the curve

$$x=\mathrm{e}^{-t}\cos t,\quad y=\mathrm{e}^{-t}\sin t,\quad t\in[0,\infty).$$



Exercise 9. Find the length of the curve $x = \ln(\sin y) \quad \pi/6 \leqslant y \leqslant \pi/3$.

7 Section 9.4

Exercise 10. (14p553) Find the area of the surface obtained by rotating the curve

$$y = 1 - x^2, \quad 0 \leqslant x \leqslant 1$$

about the y-axis.

Exercise 11. Given the curve

$$y = \frac{x^3}{6} + \frac{1}{2x} \quad 1 \leqslant x \leqslant 2.$$

1. Find the length of the curve .
2. Find the area of the surface obtained by rotating the curve about the x-axis.
3. Find the area of the surface obtained by rotating the curve about the y-axis.

Exercise 12. The curve

$$y = x^2, \quad 0 \leqslant x \leqslant 1$$

is rotated

1. about the y-axis. Find the area of the resulting surface.
2. about the y-axis. Find the area of the resulting surface.