Section 6.4: The Fundamental Theorem of Calculus

Fundamental Theorem of calculus Part 1: If f is a continuous function on $[a, b]$ then the function $g(x) = \int_a^x f(t)\mathrm{d}t$ is

- continuous on $[a, b]$,
- differentiable on (a, b),
- $g'(x) = f(x)$ for any $x \in (a, b)$.

Remarks:

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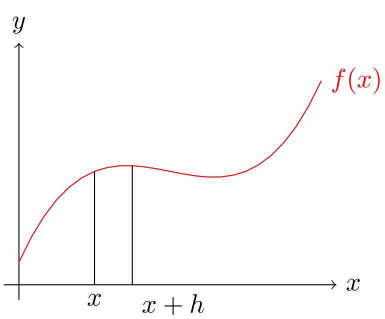

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Exercise 1. Find the derivative of $f(s) = \int_s^6 \frac{x^4 + 3x + 1}{x^2 + 4}\mathrm{d}x$.

Exercise 2. Find the derivative of $m(x) = \int_0^{\sin x} \frac{1}{\sqrt{1 - t^2}\mathrm{d}t}$ for $t \in [0, \pi/6]$.

Exercise 3. Find the derivative of the function

$$a(x) = \int_{e^{2x}}^{e^{3x}} \frac{1}{s}\mathrm{d}s.$$

Fundamental Theorem of calculus Part 2: If f is a continuous function on $[a, b]$ and F is an antiderivative of f, then

$$\int_a^b f(t)\mathrm{d}t = F(b) - F(a).$$

Remark:

Exercise 4. Find $\displaystyle\int_1^4 x^2 + \sqrt{x} - \frac{1}{x^2}\mathrm{d}x$.

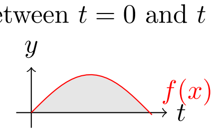
Exercise 5. Find $\displaystyle\int_{-2}^2 t^3\mathrm{d}t$.

Exercise 6. Find $\displaystyle\int_0^\pi \cos x + \sin x\mathrm{d}x$.

Exercise 7. Find the area under the curve

$$y = \sin t \cos t$$

between $t = 0$ and $t = \pi/2$.



Notation: If f is a continuous function, the definite integral $\displaystyle\int f(t)\mathrm{d}t$ means an antiderivative of f.

Exercise 8. Find

$$\int \frac{1}{3t + 2}\mathrm{d}t.$$