Section 7.2: Volume

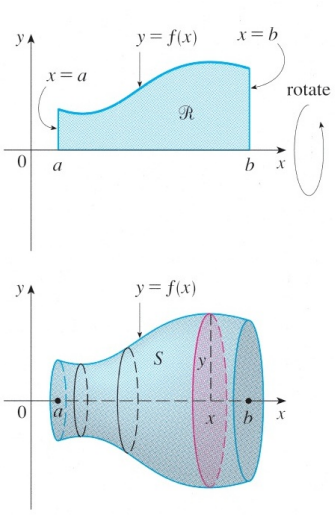
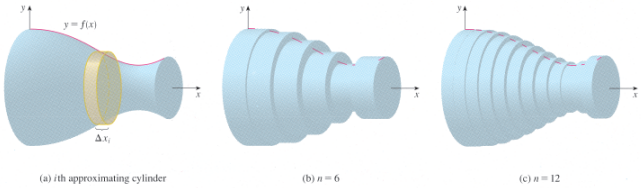


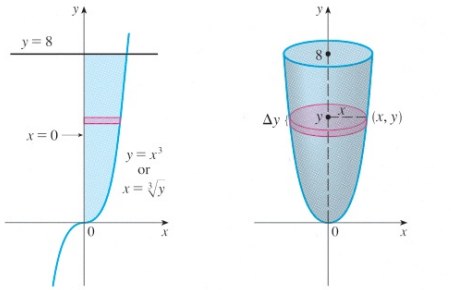
FIGURE 5



Theorem: the volume of a solid obtained by rotating the region bounded by the curves $x = a$, $x = b$, $y = 0$ and $y = f(x)$ about the x -axis is

$$V = \int_a^b \pi f^2(x) \mathrm{d}x$$

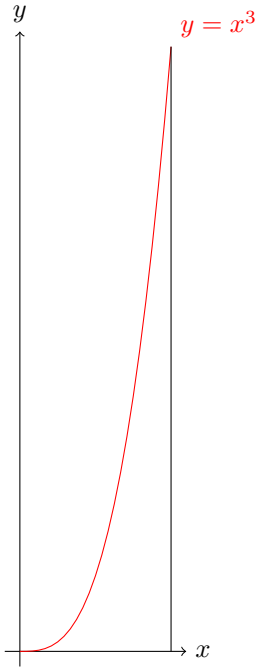
Similarly,



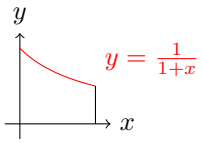
Theorem: the volume of a solid obtained by rotating the region bounded by the curves $y = a$, $y = b$, $x = 0$ and $x = f(y)$ about the y-axis is

$$V = \int_a^b \pi f^2(y) \mathrm{d}y$$

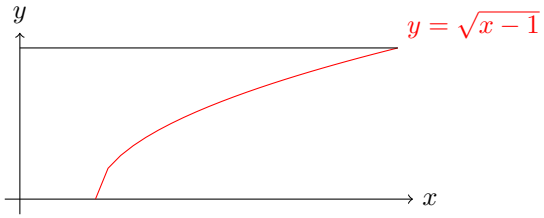
Exercise 1. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^3$, $x = 2$, and $y = 0$ about the x-axis.



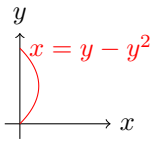
Exercise 2. Find the volume of the solid obtained by rotating the region bounded by the curves $x = 0$, $x = 1$, $y = 0$, and $y = \dfrac{1}{1+x}$ about the x-axis.



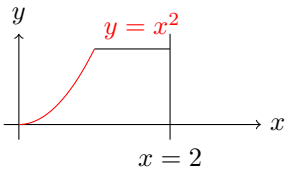
Exercise 3. Find the volume of the solid obtained by rotating the region bounded by the curves $y = \sqrt{x-1}$, $y = 0$, and $y = 2$ and, $x = 0$ about the y-axis.



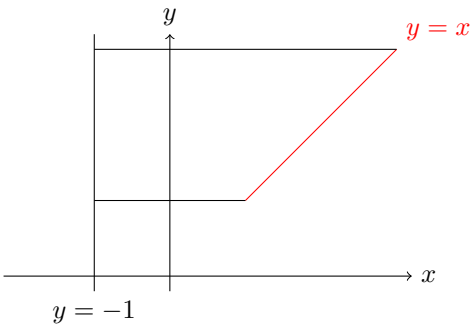
Exercise 4. Find the volume of the solid obtained by rotating the region bounded by the curves $x = y - y^2$, $x = 0$, about the y-axis.



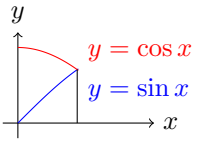
Exercise 5. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$, $y = 0$, and $y = 1$ about the axis $x = 2$.



Exercise 6. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x$, $x = 1$, $x = 3$ about the axis $y = -1$.



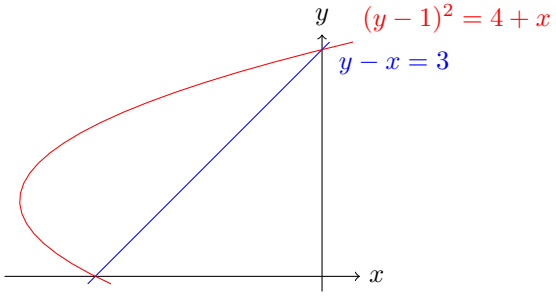
Exercise 7. Find the volume of the solid obtained by rotating the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \dfrac{\pi}{4}$ about the x-axis.



Theorem (washer method): Let S be a solid generated by rotating the region bounded by the curves $y = f(x)$, $y = g(x)$, $x = a$ and $x = b$ with $f(x) \geqslant g(x)$ on $[a, b]$ about the x axis, then

$$V = \int_a^b \pi(f^2(x) - g^2(x))\mathrm{d}x.$$

Exercise 8. Set up an integral for the volume of the solid obtained by rotating the region bounded by the curves $y - x = 3$, and $(y-1)^2 = 4 + x$ about $x = 1$.



Exercise 9. (56p72) The base of S is a circular disc with radius r. Parallel cross sections perpendicular to the base are squares. Find the volume of S

Exercise 10. The base is the parabolic region $\{(x, y)|x^2 \leqslant y \leqslant 1\}$. Cross sections perpendicular to the y-axis are equilateral triangles. Find the volume of the solid.