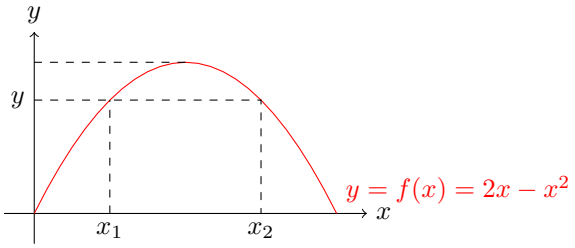
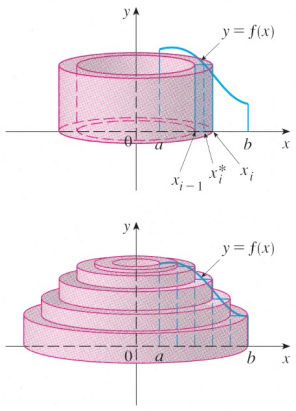
Section 7.3: Volumes by cylindric shells

Exercise 1. Find the volume of the solid obtained by rotating the area bounded by the curves $y = 2x - x^2$ and $y = 0$ about the y-axis.



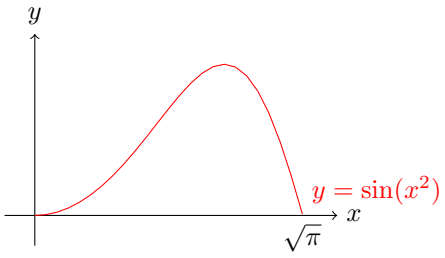


Theorem (Method of cylindric shells): The volume of a cylinder obtained by rotating the area bounded by the curves $y = f(x)$, $x = a$, $x = b$, and $y = 0$ about the y-axis is given by

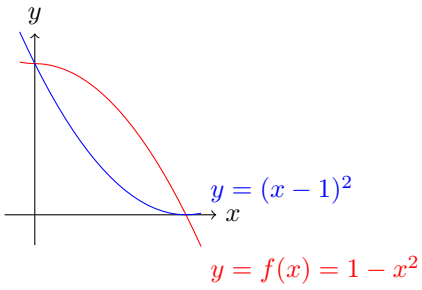
$$\int_a^b 2\pi x f(x) \mathrm{d}x$$

Remark:

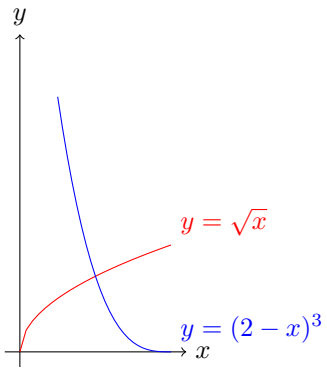
Exercise 2. Find the volume generated by rotating the region bounded by the curves $y = \sin(x^2)$, $y = 0$, $x = 0$, and $x = \sqrt{\pi}$.



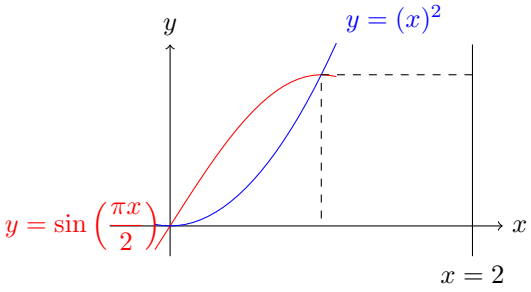
Exercise 3. Find the area generated by revolving about the y-axis the region enclosed by the curves $y = 1 - x^2$ and $y = (x-1)^2$.



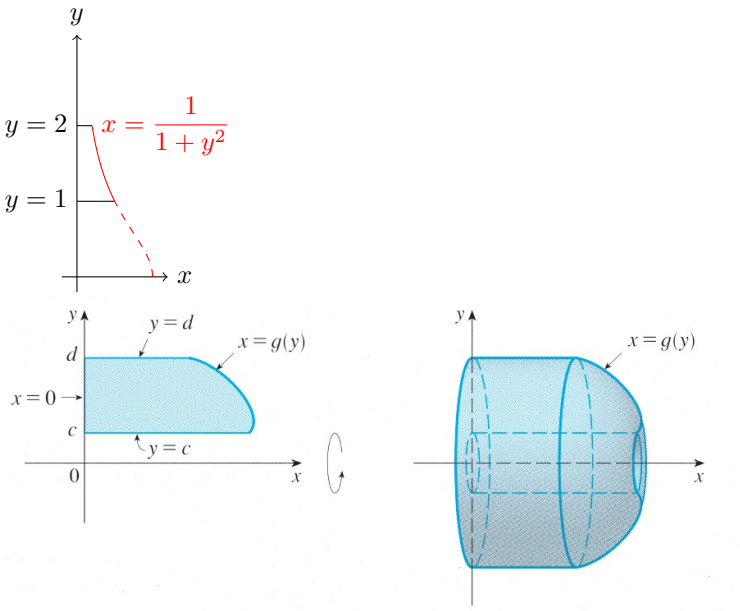
Exercise 4. Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curves $y = \sqrt{x}$, $y = 0$, $y = (x-2)^3$ about the y-axis.



Exercise 5. Find the volume of the solid obtained by rotating the region bounded by the curves $y = \sin\left(\frac{\pi x}{2}\right)$ and $y = x^2$, about the line $x = 2$



Exercise 6. Find the volume of the solid obtained by rotating the region bounded by the curves $x = \frac{1}{1+y^2}$, $y = 1$, $y = 2$, and $x = 0$.

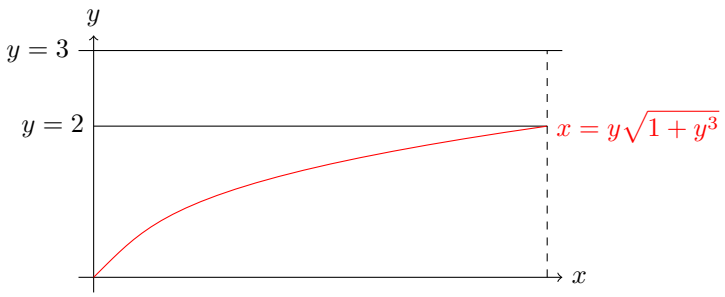


Theorem (Method of cylindric shells): The volume of a cylinder obtained by rotating the area bounded by the curves $x = g(y)$, $y = c$, $y = d$, and $x = 0$ about the x-axis is given by

$$\int_c^d 2\pi y g(y) \mathrm{d}y$$

Exercise 7. Find the volume of the solid obtained by rotating the region bounded by the curves $x = y\sqrt{1+y^3}$, $y = 0$, $y = 2$, and $x = 0$

- about the x-axis.



- about the line $y = 3$

Washer method or method of cylindric shells:		
Rotation	**about vertical lines**	**about horizontal lines**
Curves $y = f(x)$	cylindric shells	washer method
Curves $x = g(y)$	washer method	cylindric shell

Exercise 8. Use the washer method or the method of cylindric shells to find the volume obtained by rotating the area bounded by the curves $x = 4y - y^2$, and $x = 0$.

- about the line $x = 4$.

- about the x-axis.