
Section 7.5: Average value of a function

Definition: Let f be a continuous function on $[a, b]$, the **average value function** of f on the interval $[a, b]$ is

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx.$$

Exercise 1. Find the average value function of $f(t) = \cos t \sin^3 t$ on $[0, \pi/2]$.

Exercise 2. The temperature (in °F) in a certain city t hours after 9:00am was approximated by the function

$$T(t) = 80 + 14 \sin\left(\frac{\pi t}{12}\right)$$

Find the average temperature during the period of time 9:00am to 9:00pm.

Mean value Theorem for integrals: If f is a continuous function on $[a, b]$, then there exists a number c in (a, b) such that

$$\int_a^b f(x) dx = (b-a)f(c).$$

Exercise 3. (8p451) Let $f(x) = 4x - x^2$.

- Find the average value f_{ave} of f on $[0, 3]$.
- Find c such that $f(c) = f_{ave}$.
- Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f .