Section 8.4: Integration of rational functions by partial fractions

Decomposition in partial fractions method:

1. If the degree of the numerator is greater or equal to the degree of the numerator, perform a long division and work with the rest.

   Exercise 1. Evaluate $\int \frac{2x^2 + 3x + 4}{x + 1} \mathrm{d}x$

   Exercise 2. Evaluate $\int \frac{x^2}{x^2 + 1} \mathrm{d}x$

2. Factorize the denominator $Q(x)$ and guess the form of the decomposition in partial factor.

   (a) The denominator is a product of distinct linear factors $Q(x) = (x - x_1)(x - x_2) \cdots (x - x_n)$, then

   $$f(x) = \frac{P(x)}{Q(x)} = \frac{A_1}{x - x_1} + \frac{A_2}{x - x_2} + \cdots + \frac{A_n}{x - x_n}.$$

   Exercise 3. Evaluate $\int \frac{2x^2 - 5x - 24}{x^3 + x^2 - 12x} \mathrm{d}x$.

(b) The denominator $Q(x)$ is a product of linear factors, some repeated

$$f(x) = \frac{3x^2 + x}{(x-1)^2(x+1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

Exercise 4. Integrate $\displaystyle\int \frac{3x^2 + x}{(x-1)^2(x+1)} \mathrm{d}x$.

(c) The denominator contains irreducible quadratic factors, none of which is repeated:

$$f(x) = \frac{x^3 + 7x^2 - x + 1}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx + D}{x^2+1}$$

Exercise 5. Evaluate $\displaystyle\int \frac{3x^2 + 3x + 2}{(x-1)(x^2+2x+5)}$.

(d) The denominator contains irreducible repeated quadratic factors:

$$f(x) = \frac{x^3 + x^2 + 2x + 3}{(x^2+1)^3} = \frac{Ax + B}{x^2+1} + \frac{Cx + D}{(x^2+1)^2} + \frac{Ex + F}{(x^2+1)^3}$$

Exercise 6. Find the partial fraction decomposition of

$$f(x) = \frac{9}{(x^3 - 1)^2}$$

Exercise 7. Find $\displaystyle\int \frac{x^3}{(x+1)^3}\mathrm{d}x$.

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