University of Washington


**Last Name (PRINT):** ____________________
**First Name (PRINT):** ____________________

# Autumn 2014 – Linear Analysis
# Second Examination

## Instructions

1. The use of all electronic devices is prohibited. Any electronic device needs to be turn off and placed in your bag. Any textbooks or notes also need to be placed in your bag with the exception of one double sided handwritten cheat sheet.

2. Present your solutions in the space provided. Show all your work neatly and concisely. Clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

Scholastic dishonesty will not be tolerated and may result in terminating the midterm early. The work on this test is my own.

**Signature:**________________________

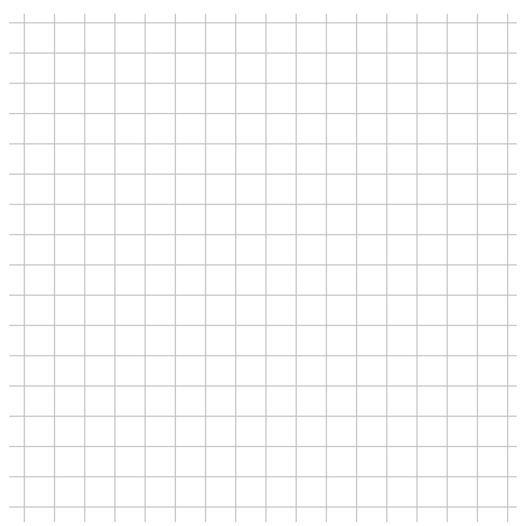
Grade:

**Exercise 1.** (5 points) Given the system

$$X' = \begin{pmatrix} -1 & -1 \\ 2 & -1 \end{pmatrix} X$$

Classify the critical point $(0, 0)$ as to type (node, proper node, improper node, spiral point, center, saddle point,...) and determine whether it is stable asymptotically stable, or unstable.

Sketch several trajectories.



**Exercise 2.** (5 points) Find the eigenvalues and eigenfunctions of the boundary problem

$$y'' + \lambda y = 0, \qquad y(0) = y(2\pi), \quad y'(0) = y'(2\pi).$$

**Exercise 3.** (5 points) Given the $2\pi$-periodic function

$$f(x) = \begin{cases} -1, & -\pi \leqslant x < 0 \\ 0, & 0 \leqslant x < \pi \end{cases}$$

$$f(x + 2\pi) = f(x)$$

1. Show that the Fourier series of $f$ is

$$\tilde{f}(x) = -\frac{1}{2} + \frac{2}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots + \frac{\sin((2n-1)x)}{2n-1} + \cdots\right)$$

2. Find the value of the sum

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{(n+1)}}{2n-1}$$

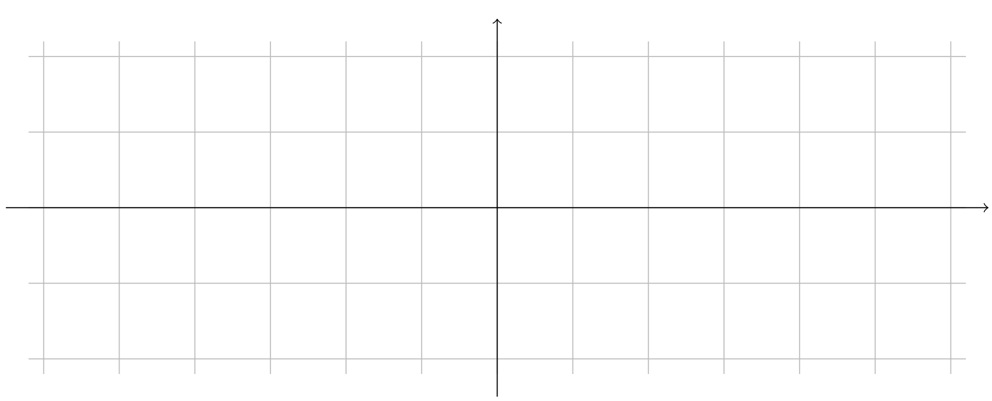
**Exercise 4.** (5 points) Given the 2-periodic function

$$f(x) = x \qquad x \in (-1, 1)$$

$$f(x) = f(x + 2)$$

1. Find the Fourier series of $f$

2. Sketch a careful graph of the function to which the Fourier series is convergent over 3 periods.



**Exercise 5.** (5 points) Use the method of separation of variables to solve the heat flow problem for a rod of length $\pi$ and diffusivity 3 and the following boundary conditions

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}, \qquad t > 0 \quad 0 < x < \pi.$$

$$u(0,t) = u(\pi,t) = 0, \qquad t > 0$$

$$u(x,0) = \sin 2x - 6\sin 5x, \qquad 0 < x < \pi.$$