University of Washington

Last Name (PRINT): ____________________

First Name (PRINT): ____________________

Autumn 2018 – Linear Analysis
Second Examination

Instructions

1. The use of all electronic devices is prohibited. Any electronic device needs to be turn off and placed in your bag. Any textbooks or notes also need to be placed in your bag with the exception of one double sided handwritten cheat sheet.
2. Present your solutions in the space provided. Show all your work neatly and concisely. Clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

Scholastic dishonesty will not be tolerated and may result in terminating the midterm early. The work on this test is my own.

Signature:________________________

Grade:

Exercise 1. (5 points) Find the eigenvalues and eigenfunctions of

$$y'' + \lambda y = 0 \qquad y'(-1) = 0, \quad y'(1) = 0$$

Exercise 2. (5 points) Find the Fourier series of the 4-periodic function f defined on $(-2, 2]$ by

$$f(x) = \sin x \quad \text{for} \quad x \in (-2, 2], \qquad f(x+4) = f(x).$$

Sketch a careful graph of the Fourier series for three periods.
Make sure specify the values at the end points.

Exercise 3. (5 points) Given the function

$$f(x) = \begin{cases} 0, & 0 < x \leqslant 2 \\ 1, & 2 < x < 3 \end{cases}$$

Find the Fourier series of the even extension of period 6.

Sketch a careful graph of the even extension of f over 3 periods.
Make sure specify the values at the end points.

Exercise 4. (5 points) Use the method of separation of variables to replace the partial differential equation with boundary conditions

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} + 3u = 0, \qquad 0 < x < 1, \quad 0 < t < 1,$$

$$u(t,0) = 0, \qquad \frac{\partial u}{\partial x}(t,1) = 0$$

$$u(0,x) = 0.$$

by a system of 2 ordinary differential equations with boundary conditions.

Give the system of equations and the corresponding boundary conditions. Do not solve the problem.

Exercise 5. (5 points) Consider a rod of π cm whose initial temperature is given by

$$u(x,0) = 1 - \cos(3x) + 2\cos(5x), \qquad x \in [0,\pi]$$

Assume that both ends of the bar are insulated:

$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0, \qquad t > 0.$$

Suppose that $\alpha^2 = 0.25$ cm^2/s.

Find the temperature $u(x,t)$.

I need to see the reasoning leading to your solution.

You do not need to solve in great detail classic eigenvalue problems as long as your conclusions are correct.

You need to find the coefficients.

Possibly useful trigonometric formula

$$\cos a \cos b = \frac{1}{2}(\cos(a-b) + \cos(a+b))$$
$$\cos a \sin b = \frac{1}{2}(\sin(a+b) - \sin(a-b))$$
$$\sin a \sin b = \frac{1}{2}(\cos(a-b) - \cos(a+b))$$
$$\sin a + \sin b = 2\sin(\frac{a+b}{2})\cos(\frac{a-b}{2})$$
$$\sin a - \sin b = 2\cos(\frac{a+b}{2})\sin(\frac{a-b}{2})$$
$$\cos a + \cos b = 2\cos(\frac{a+b}{2})\cos(\frac{a-b}{2})$$
$$\cos a - \cos b = -2\sin(\frac{a+b}{2})\sin(\frac{a-b}{2})$$