University of Washington

Last Name (PRINT): ____________________
First Name (PRINT): ____________________

Spring 2017 – Linear Analysis
Second Examination

Instructions

1. The use of all electronic devices is prohibited. Any electronic device needs to be turn off and placed in your bag. Any textbooks or notes also need to be placed in your bag with the exception of one double sided handwritten cheat sheet.
2. Present your solutions in the space provided. Show all your work neatly and concisely. Clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

Scholastic dishonesty will not be tolerated and may result in terminating the midterm early. The work on this test is my own.

Signature:____________________

Grade:

Exercise 1. (5 points) Either solve the given boundary value problem or show that it has no solution.

$$y'' - y = 0, \qquad y'(0) = 1, \quad y(L) = 0$$

The answer may depend on the value of L.

Exercise 2. (5 points) Find the eigenvalues and eigenfunctions of

$$y'' + \lambda y = 0 \qquad y(-1) = 0, \quad y'(1) = 0$$

Exercise 3. (5 points) Given the function

$$f(x) = \begin{cases} 1, & 0 < x \leqslant 1 \\ 0, & 1 < x < 3 \end{cases}$$

Find the Fourier series of the odd extension of period 6.

Sketch a careful graph of the odd extension over three periods.
Make sure specify the values at the end points.

Sketch a careful graph of the Fourier series of the odd extension over 3 periods.

Exercise 4. (5 points) Use the method of variation of variables to replace the partial differential equation with boundary conditions

$$\sqrt{1+t}\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \qquad 0 < x < 1, \quad t > 0,$$

$$u(t,0) = 0, \qquad \frac{\partial u}{\partial x}(t,1) = 0$$

$$u(0,x) = 1 - x^2.$$

by a system of 2 ordinary differential equations with boundary conditions.

Do not solve the problem, I am expecting a pair of two ordinary differential equations and a set of boundary conditions.

Exercise 5. (5 points)

Let a rod of length 50 cm be initially at the uniform temperature of 30°C. Suppose that at time $t = 0$, the end $x = 0$ is cooled to 10°C while the end $x = 50$ is heated to 60°C, and both are thereafter maintained at those temperature.

Assume the diffusivity constant $\alpha^2 = 2$.

Find the temperature distribution $u(x, t)$ in the rod at any time t.

I am expecting the reasoning that leads you to the answer.

If you find an eigenvalue problem, you do not need to solve it in great detail and may jump directly to the eigenvalues and eigenfunctions.

You may leave the coefficients in terms of explicit definite integrals and not evaluate them. I need to see a precise expression of the integrals that I can compute.